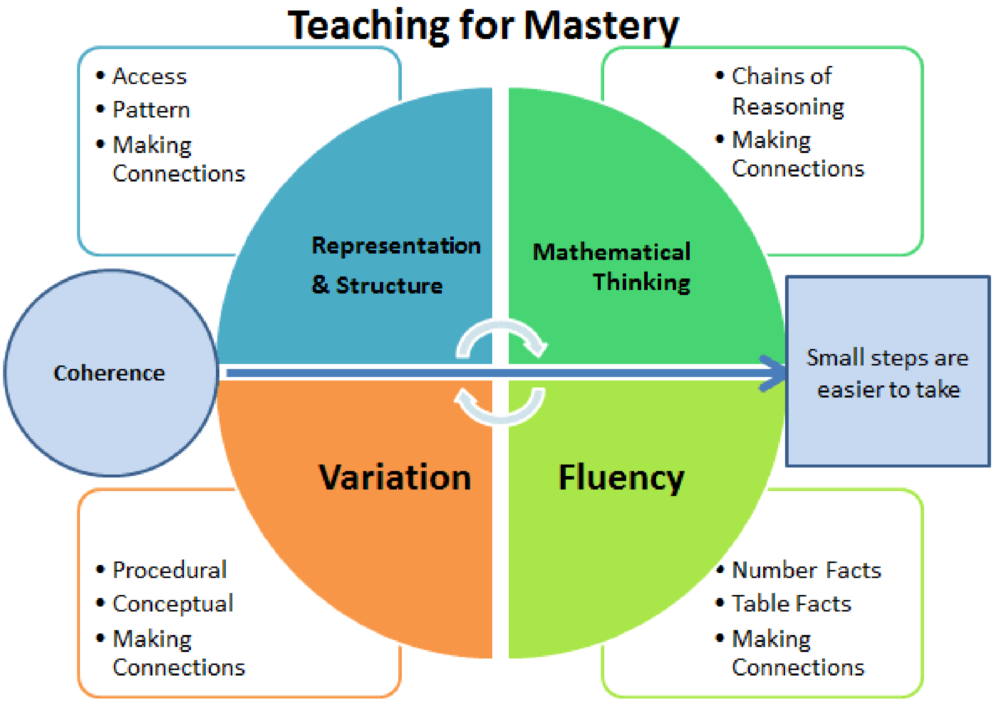
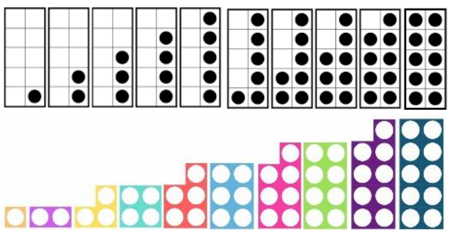
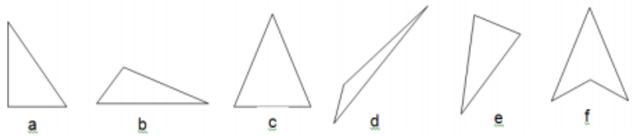
The NCETM have drawn five big ideas[7] from research evidence that underpin teaching for mastery in maths.



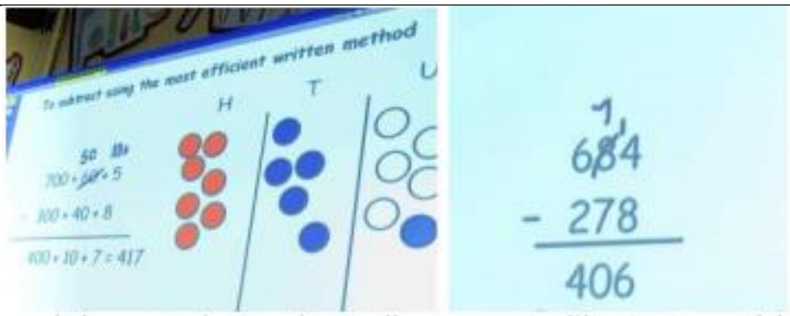
The NCETM provides an overview of each of the big ideas in mastery.

Coherence	
Messages	Example
- Small steps are easier to take. - Focusing on one key point each lesson allows for deep and sustainable learning. - Certain images, techniques and concepts are important pre-cursors to later ideas. Getting the sequencing of these right is an important skill in planning and teaching for mastery.	Before teaching the written algorithm for subtraction, 4 7 - 3 8 pupils need to be able to: → be fluent in their number facts for single digit numbers, → have a good understanding that 47 can be partitioned into 40 and 7 or 30 and 17, → understand that 40 can be thought of as 4 tens and → understand that 3 tens and 4 tens make 7 tens and that this is the same as 30 and 40 make 70.
Representations and structure	
Messages	Example
- The representation needs to pull out the concept being taught, and in particular, the key difficult point. **It exposes the structure.** - In the end, the children need to be able to do the maths without the representation. - **A stem sentence** describes the representation and helps the children move to working in the	Here are two representations for numbers within 10: the tens frame and Numicon.  Both are very helpful concrete and pictorial representations of number but, crucially, they are representing different structures. The tens frame is

<table>
<tr><td>abstract (“ten tenths is equivalent to one whole”).
- Pattern and structure are related but different: children may have seen a pattern without understanding the structure which causes that pattern.</td><td>accentuating and drawing attention to the ‘5 and a bit’ structure of numbers, whereas Numicon draws attention to the odd/even structure. Both images support seeing the complement to 10 (i.e. what needs to be added to make 10). They offer different (equally important) ways of thinking about the structure which in turn influence the ways children might transform, compare and combine numbers when calculating.</td></tr>
<tr><td colspan="2">Variation</td></tr>
<tr><td>Messages</td><td>Example</td></tr>
<tr><td>- The central idea is to highlight the essential features of a concept or idea through varying the non-essential features.
- When giving examples of a concept, it is useful to add variation to emphasise what it is (as varied as possible) and what it is not.
- When constructing a set of activities/questions, it is important to consider what connects the examples – what mathematical structure is being highlighted.
- Variation is <u>not</u> the same as variety – careful attention needs to be paid to what aspects are being varied and for what purpose.</td><td><u>Procedural variation</u> = step by step how we proceed through the exercise / the lesson.
<table>
<tr><td>58 – 24 = ___</td><td>36 – 25 = ___</td><td>53 – 22 = ___</td><td>49 – 24 = ___</td></tr>
<tr><td>57 – 25 = ___</td><td>46 – 24 = ___</td><td>64 – 23 = ___</td><td>48 – 25 = ___</td></tr>
<tr><td>56 – 26 = ___</td><td>56 – 23 = ___</td><td>75 – 24 = ___</td><td>47 – 26 = ___</td></tr>
</table>
This variation draws attention to the relationship between the two numbers in a subtraction and encourages some reasoning to explain why the answers change in the way they do.

<u>Conceptual variation</u> = varying the representation to draw out the essence of the concept.


To get a sense of what a triangle is, learners need to see examples of triangles which show all aspects being varied. If most triangles are shown with one side as a horizontal base and the vertex pointing upwards, this feature might be over-generalised and pupils might think that d or e are not triangles. It is also important to give non-examples, as in f and to discuss why this is not a triangle.</td></tr>
<tr><td colspan="2">Fluency</td></tr>
<tr><td>Messages</td><td>Example</td></tr>
<tr><td>- Fluency encompasses a mixture of efficiency, accuracy and flexibility.
- Quick and efficient recall of facts and procedures is important in order for learners’ to keep track of sub problems, think strategically and solve problems. – Fluency demands the flexibility to move between different contexts and representations of mathematics, to recognise relationships and make connections and to make appropriate choices from a whole toolkit of methods, strategies and approaches.</td><td>Quick and accurate recall of all multiplication facts up to 12 × 12 is important in order to free working memory to see the big picture and make decisions about when to use this knowledge to solve certain problems.
However, if a pupil only knows these facts as an unconnected collection of memorised phrases and does not know
- that 8 × 6 is the same as 6 × 8 or twice 4 × 6 or 12 less that 10 × 8; or
- that know the connection between 6 × 8 and 16 × 8 or 6 × 80 or 0.6 × 8
- or when faced with a problem of finding how many books are in a bookcase with 8 shelves and 6 books on each shelf, does not know what mathematics to use…
then they have not attained fluency.</td></tr>
<tr><td colspan="2">Mathematical thinking</td></tr>
<tr><td>Messages</td><td>Example</td></tr>
<tr><td>- Mathematical thinking is central to deep and sustainable learning of mathematics.</td><td>Asking “what’s the same and what’s different?” in a range of situations prompts and promotes mathematical thinking</td></tr>
</table>

- Taught ideas that are understood deeply are not just ‘received’ passively but worked on by the learner. They need to be thought about, reasoned with and discussed. - Mathematical thinking involves looking for patterns in order to discern structure, looking for relationships and connecting ideas and reasoning logically, explaining, conjecturing and proving.	 Asking pupils to explain, convince, draw diagrams to illustrate an idea or strategy and reason and conjecture as a natural part of all activity in the mathematics classroom supports deep and sustainable learning.